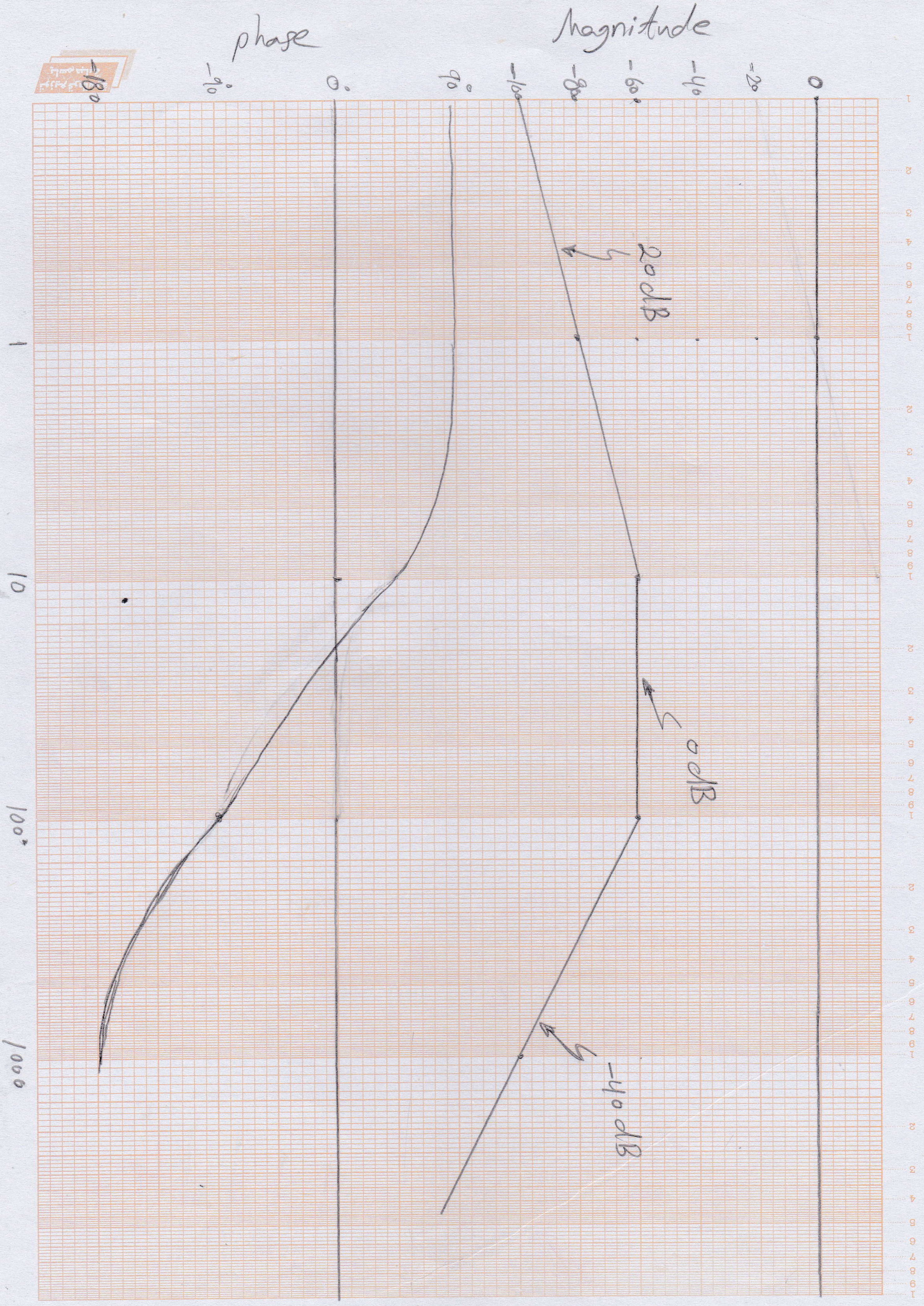
EX sketch the Bode plot of the transfer function

$$H(s) = \frac{10\,s}{(s+10)\,(s+100)^2}$$

$$H(j\omega) = \frac{\cancel{10}\; j\omega}{100^2 \times \cancel{10}\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{100}+1\right)^2}$$

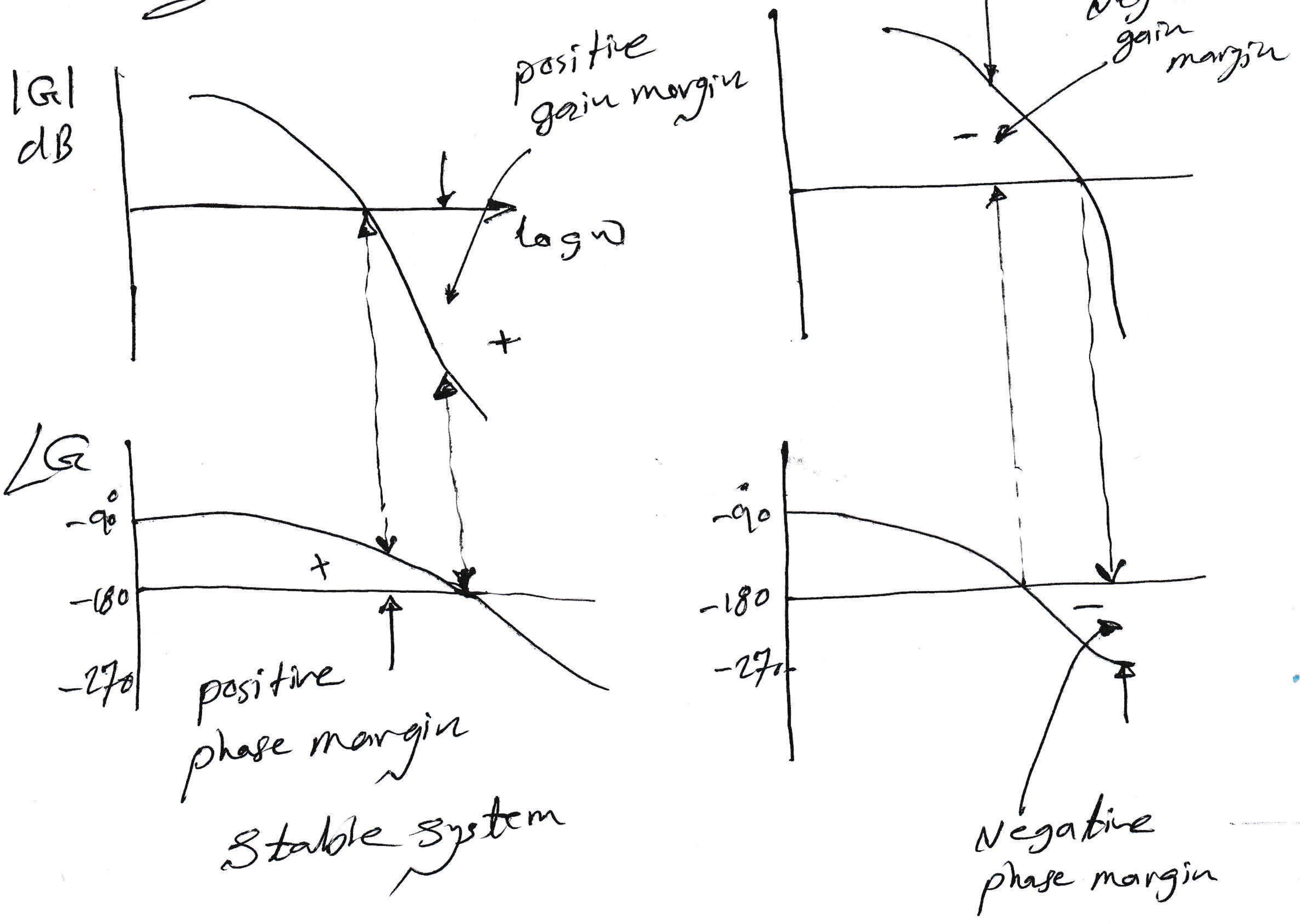
$$H(j\omega) = \frac{(0.01)^2\; j\omega}{\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{100}+1\right)^2}$$

$$20\log(0.01)^2 = -80.$$


phase
Magnitude
0
-20
-40
-60
-80
-100
90°
0°
-90°
-180°
1
10
100
1000
20 dB
0 dB
-40 dB

8-2 Gain Margin and phase Margin

Gain and phase Margin are useful techniques in assessing the stability of feedback systems.



$\phi = \angle G(j\omega_c)H(j\omega_c) + 180°$

$\angle G(j\omega_c)H(j\omega_c) > 180° \Rightarrow$ PM is stable, otherwise is unstable.

- When the phase angle curve crosses the $-180°$ line, the frequency is read from the Bode phase angle curve, this frequency is ω_c. If at ω_c, the gain is positive, it means that $|G(j\omega)H(j\omega)| > 1$, and the system is unstable

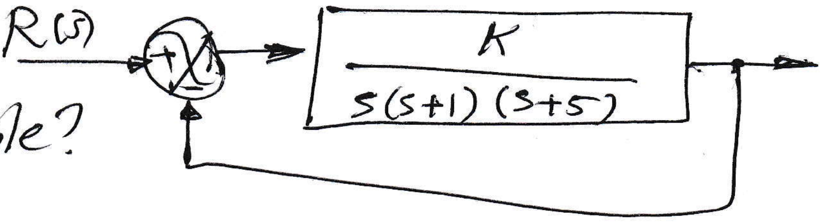
$|G(j\omega)H(j\omega)| = 1 \Rightarrow$ the system is oscillatory.

$|G(j\omega)H(j\omega)| < 1 \Rightarrow$ the system is stable.

EX Find Gain and phase margin for the system below.

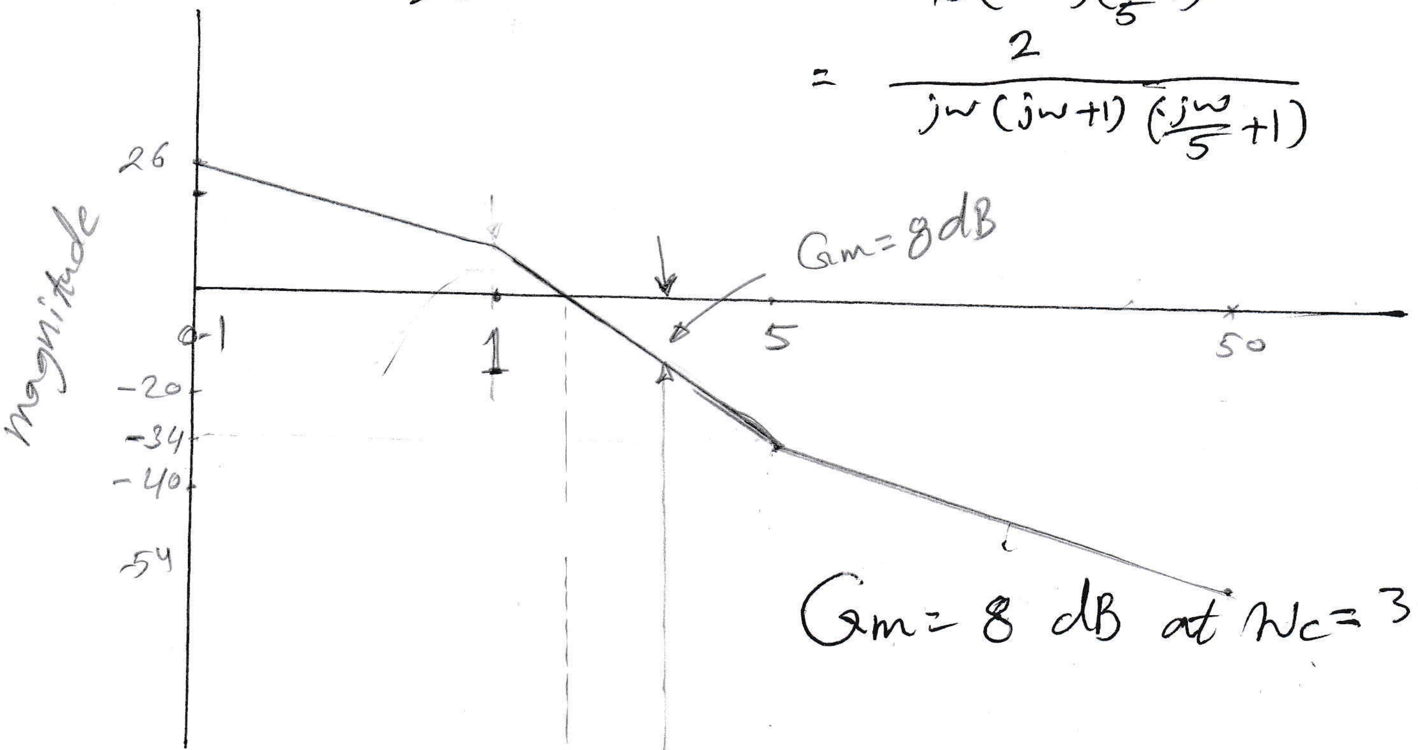
for $K = 10$.

Is the system stable?

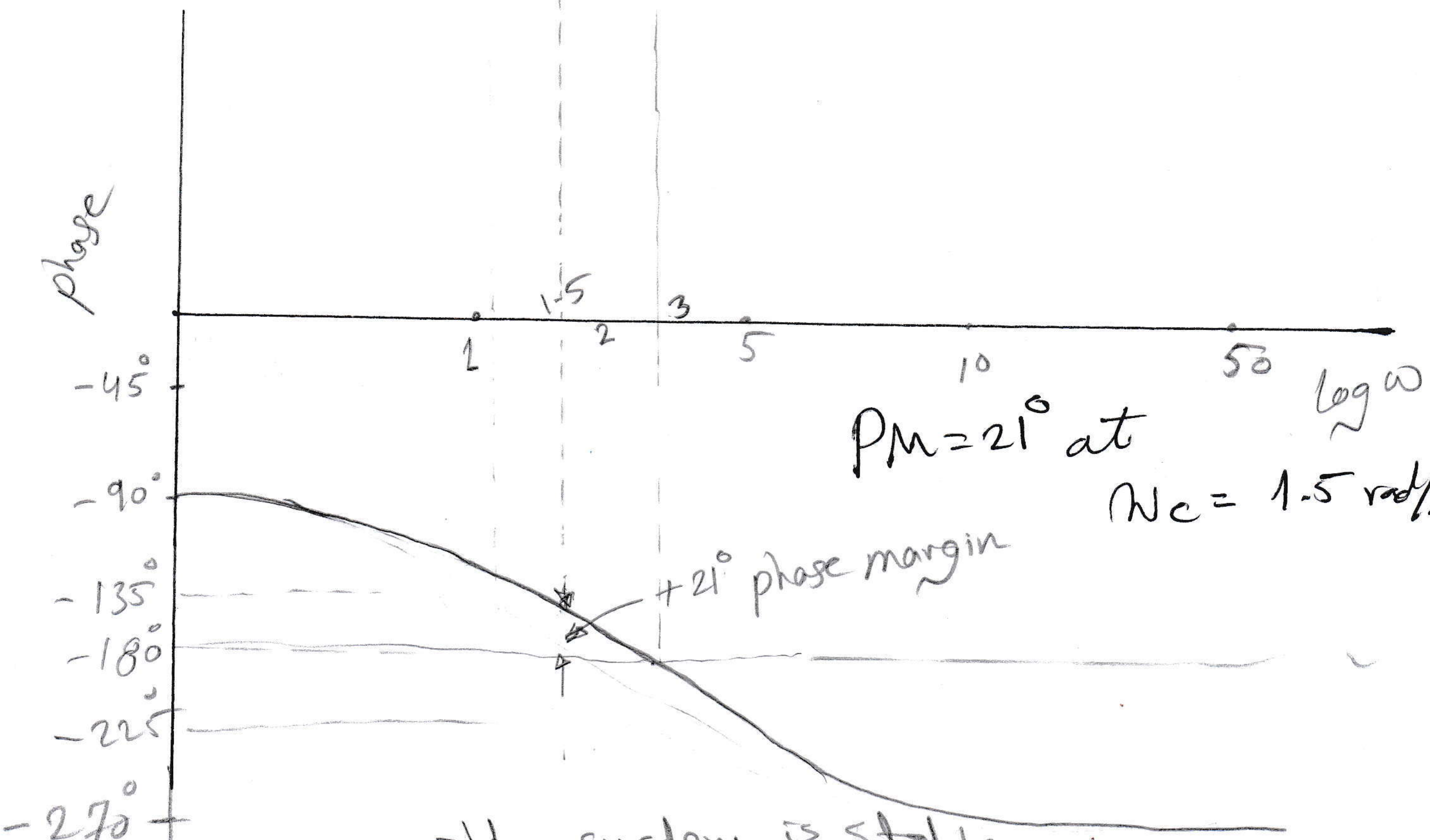


$$G(s) = \frac{10}{s(s+1)(s+5)} = \frac{10}{5\,s(s+1)\left(\frac{s}{5}+1\right)}$$

$$= \frac{2}{j\omega(j\omega+1)\left(\frac{j\omega}{5}+1\right)}$$



Gm = 8 dB at $\omega_c = 3$ rad/sec



PM = 21° at $\omega_c = 1.5$ rad/sec

the system is stable

Ex Do the same example but when $K = 100$, and find GM and PM.

EX Determine the gain and phase margin for the unity feedback system with $K = 1$.

$$G(s) = \frac{K}{s(0.5s+1)(0.05s+1)}$$

Determine the value of K for obtaining
(a) gain margin of 20 db.
(b) phase margin of 40°.

from figure, we got

$GM = 26$ db at $K=1$ with $w_{gc} = 0.9$ rad/sec
$PM = 63^\circ$ at $K = 1$ wit $w_{pc} = 6$ rad/sec

(a) we need gain margin at 20 dB, then

$26 - 20 = 6$ db

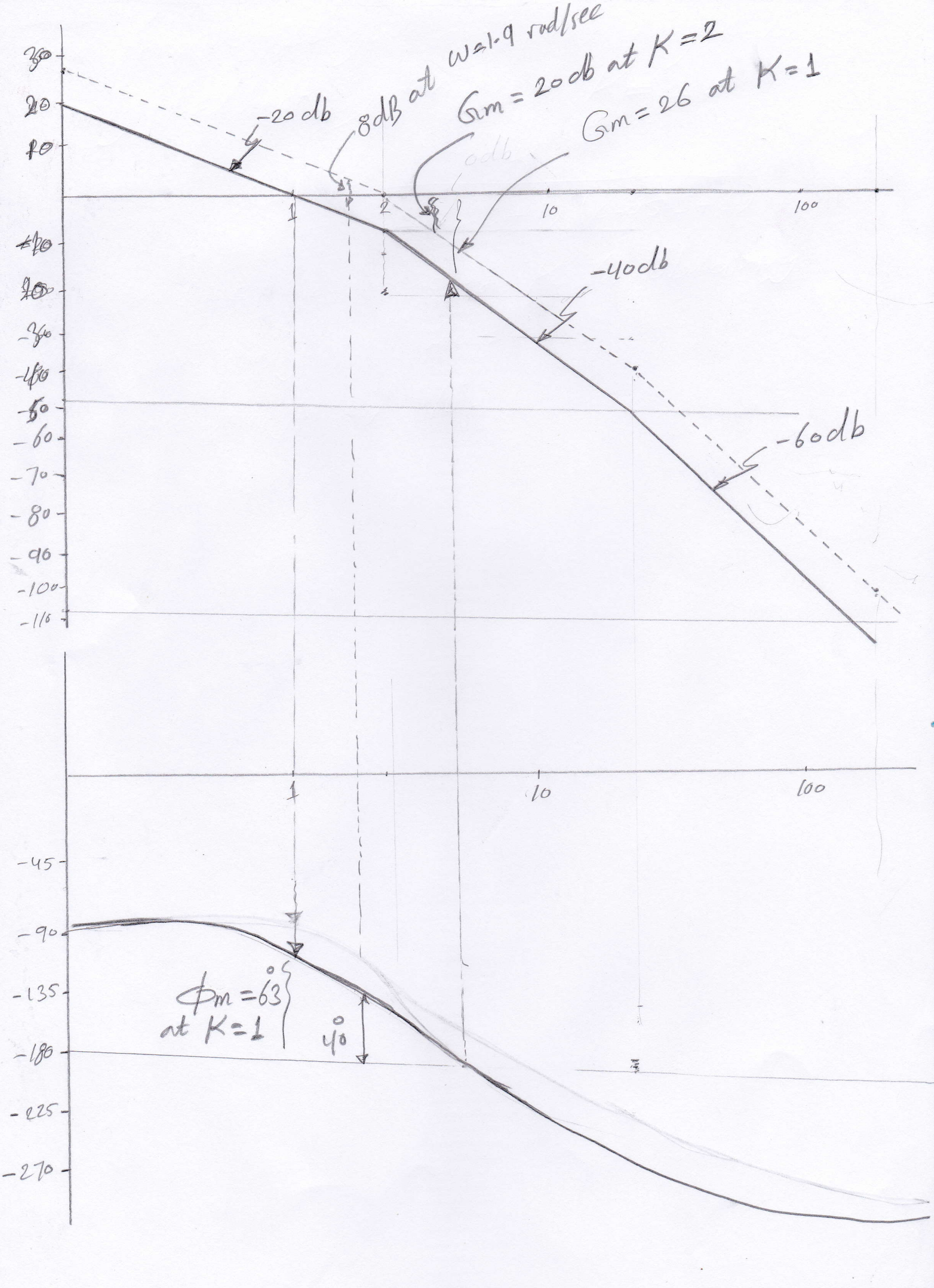
$20 \log K = 6 \Rightarrow \log K = \frac{6}{20} \Rightarrow \log K = 0.3$

$K = 1.995 \simeq 2$

b) To get $PM = 40^\circ$, then locate the frequency at which the phase curve has an angle of $(-180+40) = -140^\circ$

find the gain this frequency.
the magnitude at the frequency $w = 1.9$ rad/sec is 8 dB

$\therefore 20 \log K = 8 \Rightarrow K = 2.51$



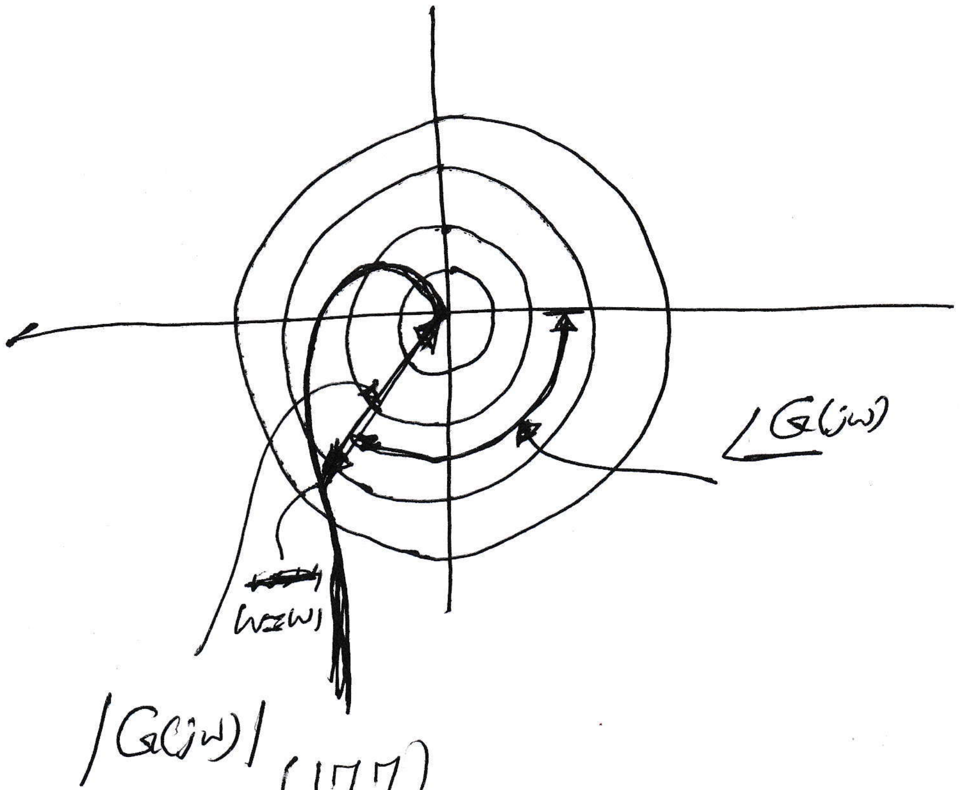
8dB at ω=1.9 rad/sec
Gm = 20db at K=2
Gm = 26 at K=1
-20db
-40db
-60db
1
2
10
100
30
20
10
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-45
-90
-135
-180
-225
-270
Φm = 63° at K=1
40°

8-3 Polar plots:

There is another graphical representation of sinusoidal transfer function $G(j\omega)$. For a given value of ω, $G(j\omega)$ is a complex number and its has magnitude and angle,

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$
$$= M \angle \theta$$

As ω is changed from 0 to ∞ this vector will change in magnitude and phase angle. This curve or figure is known as "polar plot" of the given transfer function. This plot is useful in determining the stability of the system in frequency domain, using Nyquist stability criterion



EX Draw the polar plot of $G(s) = \frac{1}{1+T_1 s}$

the Sinusoidal transfer function is given by

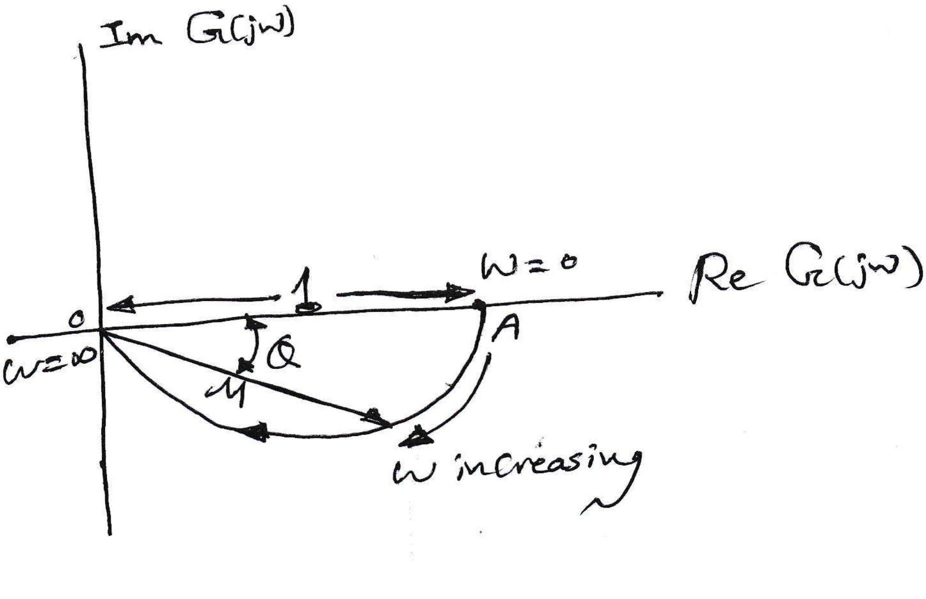
$$G(j\omega) = \frac{1}{1+j\omega T_1}$$

$$= \frac{1}{\sqrt{1+\omega^2 T_1^2}} \angle -\tan^{-1}\omega T_1$$

$$= M \angle \theta$$

- at $\omega = 0 \Rightarrow M = 1, \quad \theta = 0^\circ$
- at $\omega = \infty \Rightarrow M = 0, \quad \theta = -90^\circ$

For any $0 \le \omega \le \infty$, $M \le 1$, $0 \le \theta \le -90$.



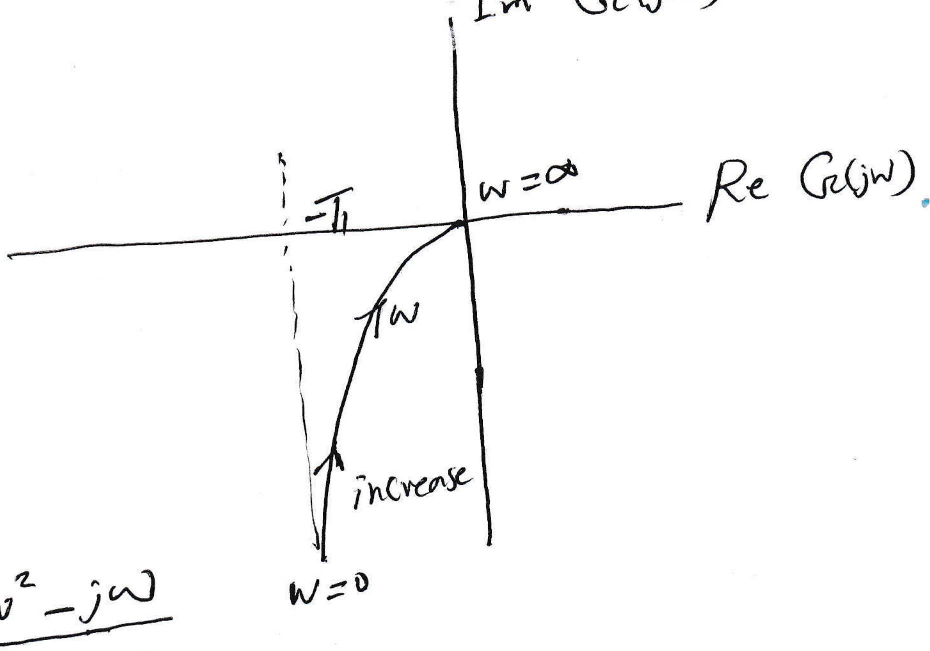
EX Draw the polar plot of $G(s) = \frac{1}{S(1+T_1 S)}$

$$G(j\omega) = \frac{1}{j\omega(1+T_1 j\omega)}$$

By this method

$$= \frac{1}{\omega\sqrt{1+\omega^2 T^2}} \angle -90 - \tan^{-1} \omega T_1$$

- at $\omega = 0 \Rightarrow M = \infty$, $\angle -90^\circ$
- at $\omega = \infty \Rightarrow M = 0$, $\angle -180^\circ$



OR by this method.

$$\frac{1}{j\omega - T_1 \omega^2}$$

$$= \frac{1}{-T_1\omega^2 + j\omega} * \frac{-T_1\omega^2 - j\omega}{-T_1\omega^2 - j\omega}$$

$$= \frac{-T_1\omega^2 - j\omega}{+T_1^2\omega^4 + \omega^2} = \frac{-T_1\omega^2}{\omega^2(T_1^2\omega^2+1)} - j\frac{\omega}{\omega^2(T_1^2\omega^2+1)}$$

$$= \frac{-T_1}{T_1^2\omega^2+1} - j\frac{1}{\omega(T_1^2\omega^2+1)} \Rightarrow |G(j\omega)| = \sqrt{\left(\frac{T_1}{T_1^2\omega^2+1}\right)^2 + \left(\frac{1}{\omega(T_1^2\omega^2+1)}\right)^2}$$

$$\angle\theta = -\tan^{-1}\frac{\omega^2 T_1^2 + 1}{\omega^2 T_1(\omega^3 T_1^2 + 1)}$$

(179)

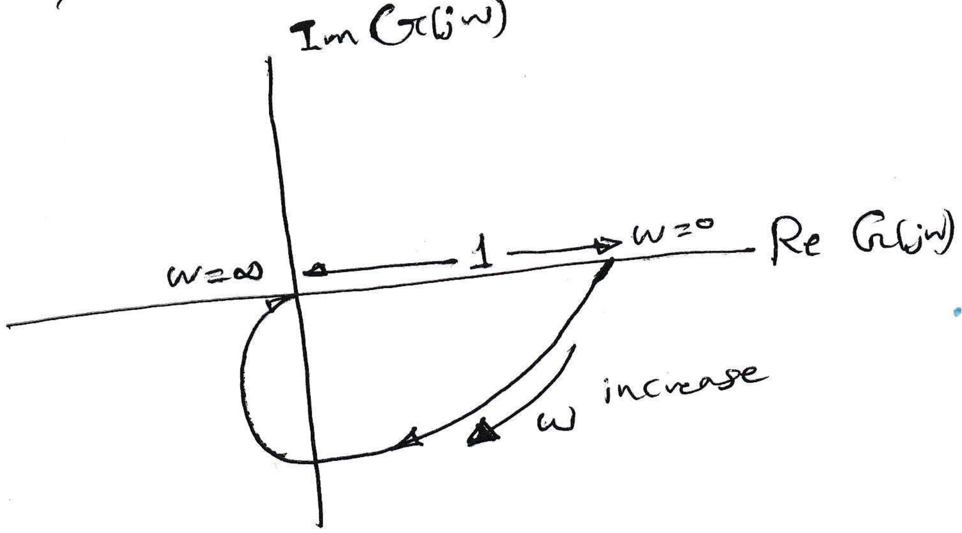
Ex Draw the polar plot of $G(s) = \dfrac{1}{(1+T_1 s)(1+T_2 s)}$

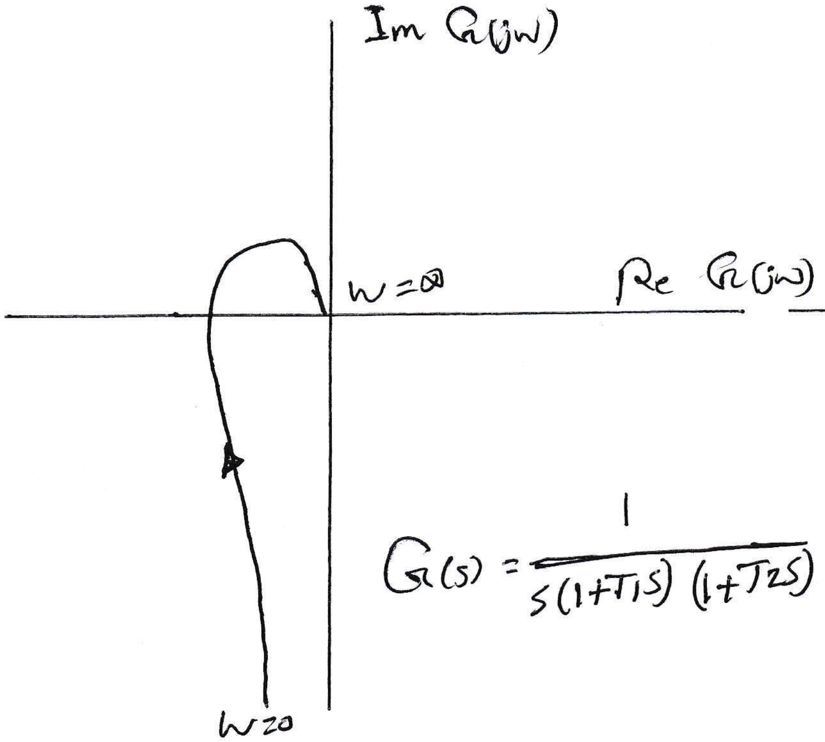
$$M = \frac{1}{\sqrt{(1+T_1^2 \omega^2)}\sqrt{(1+T_2^2 \omega^2)}}$$

$$\theta = -\tan^{-1} T_1 \omega - \tan^{-1} T_2 \omega$$

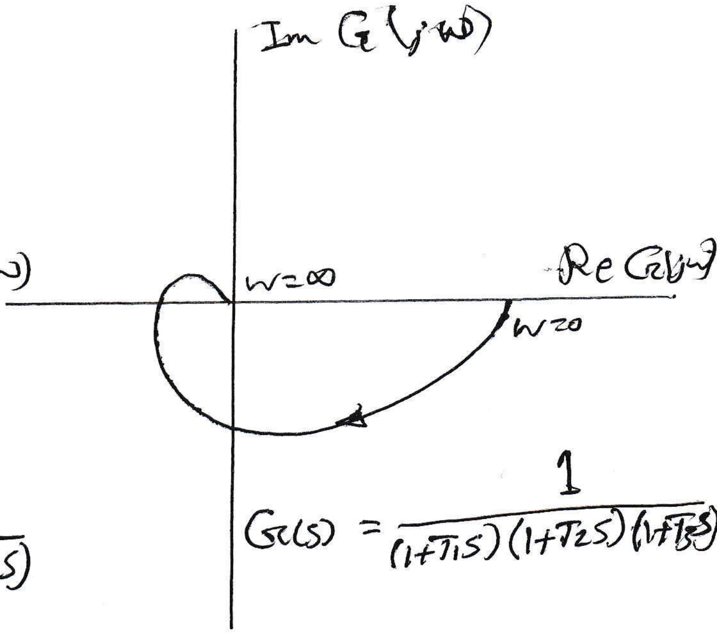
$\omega = 0 \Rightarrow M = 1 \quad , \theta = 0$

$\omega = \infty \Rightarrow M = 0 \quad , \theta = -180^\circ$

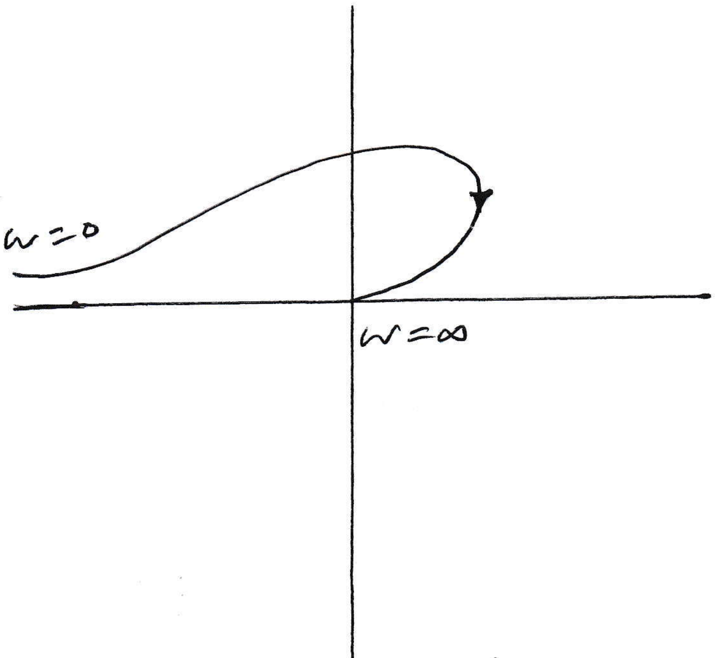




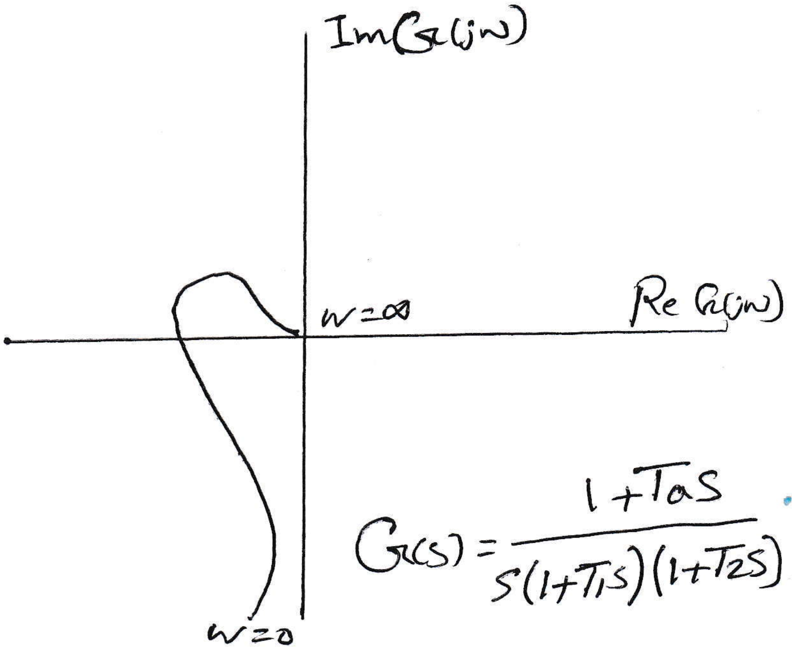
$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$$



$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)}$$



$$G(s) = \frac{1}{s^2(1+T_1s)(1+T_2s)}$$



$$G(s) = \frac{1+T_as}{s(1+T_1s)(1+T_2s)}$$

$$T_a < \frac{T_1 T_2}{T_1+T_2}$$

8-4 Nyquist Criterion :-

For a feedback control system with open loop transfer function given by :-

$$G(s)H(s) = \frac{K(s+Z_1)(s+Z_2)\cdots(s+Z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} \quad m \leq n.$$

$$= K\,\frac{\prod_{i=1}^{m}(s+Z_i)}{\prod_{j=1}^{n}(s+P_i)} \quad \text{------- (1)}$$

The characteristic equation is given by

$$D(s) = 1 + G(s)H(s) = 1 + \frac{K\prod_{i=1}^{m}(s+Z_i)}{\prod_{j=1}^{n}(s+P_j)} = 0$$

$$= \frac{\prod_{j=1}^{n}(s+P_j) + K\prod_{i=1}^{m}(s+Z_i)}{\prod_{j=1}^{n}(s+P_j)} = 0 \quad \text{------ (2)}$$

$$D(s) = \frac{(s+Z_1')(s+Z_2')\cdots(s+Z_n')}{\prod_{j=1}^{n}(s+P_j)} = 0 \quad \text{---- (3).}$$

it can be seen,

i) the poles of open loop system in eq(1) and poles of D(s) are the same in eq (3).

ii) the roots of the characteristic equation in (3), D(s) = 0 are the zeros of D(s) is given by $-Z_1', Z_2', \ldots$

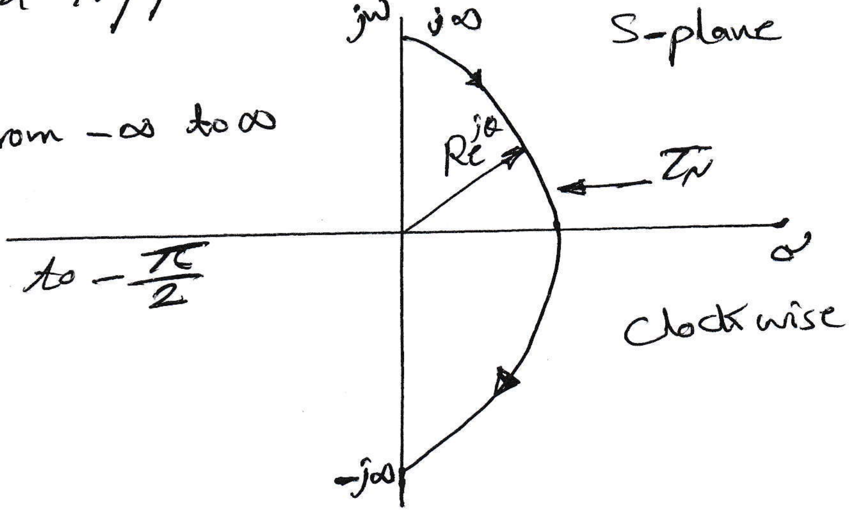
iii) the closed loop system will be stable if all the poles of the closed loop system, or all the roots of the characteristic equation lie in the left half of s-plane

8-4-1 Nyquist Countour :-

Let us consider a closed counter, T_N which encloses the entire right half of S-plane as shown in figure below. This counter is: known as a "Nyquist Countour".

$S = j\omega$ and ω varies from $-\infty$ to ∞

θ varies from $+\frac{\pi}{2}$ to 0 to $-\frac{\pi}{2}$



8-4-2 Nyquist Stability Criterion :-

If $D(s) = \dfrac{\prod_{i=1}^{n}(s+Z_i)}{\prod_{j=1}^{m}(s+P_i)}$ is plotted for values of S on the nyquist countour, the $D(s)$ plane contour will encircle the origin N times in the counter clockwise direction, where

$$N = P - Z$$

P: number of poles of $D(s)$ or the number of open loop poles in the right half of S-plane (R.H.S)

Z: Number of Zeros of $D(s)$ or the number of closed loop poles in the R.H.S.

If the closed loop System is stable,

$$Z = 0, \Rightarrow N = P$$

the number of counter clockwise encirclements of origin by the D(s) countour must be equal to the number of open loop poles in the right half of S-plane. If the open loop system is stable, there are no poles of G(s) H(s) in the RHS and hence,

$$P = 0$$

∴ For stable closed loop system,

$$N = 0$$

<u>EX</u> Obtain the Nyquist plot of a system whose open loop transfer function G(s) H(s) is given by

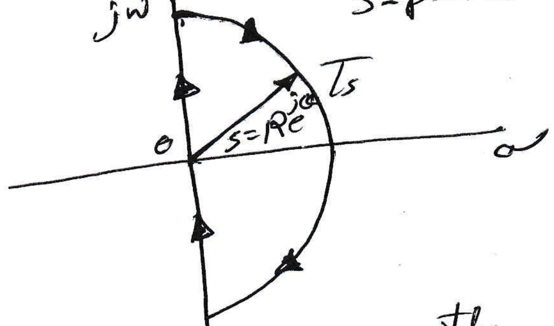
$$G(s)\,H(s) = \frac{10}{(s+2)(s+4)}$$

- $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{10}{(j\omega+2)(j\omega+4)}$$

$\omega = 0 \Rightarrow M = 1.25, \ \theta = 0°$

$\omega = \infty \Rightarrow M = 0, \ \theta = -180°$



G(s) H(s) over the contour T_s.